

Parity Violating Gravitational Coupling of Electromagnetic Fields

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A manifestly gauge invariant formulation of the coupling of the Maxwell theory with an Einstein Cartan geometry is given, where the space time torsion originates from a massless Kalb-Ramond field augmented by suitable $U(1)$ Chern Simons terms. We focus on the situation where the torsion violates parity, and relate it to earlier proposals for gravitational parity violation.

I. INTRODUCTION

In Einstein-Maxwell theory, it is well-known that the electromagnetic field-strength $F_{\mu\nu}$, defined as the generally-covariant curl of the four-potential A_μ , reduces to the flat space expression on account of the symmetric nature of the Christoffel connection [1]. However, when gravitation is taken to be described instead by Einstein-Cartan theory, i.e., a theory where the connection has an antisymmetric piece (known as spacetime torsion), the situation changes quite drastically, because the electromagnetic field strength, again defined through the covariant curl, is no longer gauge invariant [2]. As we shall argue later, the torsion tensor must necessarily obey this invariance and as such cannot be used to compensate for the loss incurred in the field strength tensor. Since electric and magnetic fields are measurable quantities irrespective of the concomitant existence of a curved background geometry, this breakdown of gauge invariance is not acceptable. Thus, one is led to infer that a coupling of Maxwell electromagnetism to Einstein-Cartan theory through the minimal coupling prescription is not possible. Attempts have been made to go beyond the minimal coupling prescription [3] - [5] by coupling a certain gauge non-invariant antisymmetric three-tensor constructed out of the electromagnetic gauge potential (now known as the Chern Simons 3-form) to a of torsion which is the (*Hodge*-) dual of the derivative of a scalar field. However, this procedure violates the Einstein Equivalence Principle if, as in [3], the scalar field is stipulated to be a function of the gravitational field alone. Alternatively, the origin of the scalar field remains unclear.

Recently, one of us (SS), in collaboration with Mukhopadhyaya [6] have explored the possibility of inducing gravitational parity violation through incorporation of such violation in the torsion tensor itself. In this manner, one obtains parity violating actions for pure gravity as well as for a host of matter couplings. However, in that paper, the coupling of gravity to non-gravitational fields is achieved through the minimal coupling prescription, and, as such, for reasons mentioned above, cannot be generalized to include electromagnetism. One of the arenas for observable gravitational parity violation might be the anisotropies of the Cosmic Microwave Background Radiation (CMBR) [7], induced by parity violating gravito-electromagnetic interactions at a more fundamental level. Therefore, it is of utmost importance to ascertain how the Maxwell field couples to the Einstein-Cartan system, possibly exhibits parity violation in the CMBR anisotropies, and yet manifestly remains gauge invariant.

There do exist phenomenological models leading to observable gravitational parity violations [8]. Indeed, the spin precession of neutrons in a gravitational field can lead to parity violating effects which may soon become observable through NMR studies [9]. Unfortunately, such phenomenological models do not ensue from the Einstein Cartan theory; in fact, low energy theorems based on one-graviton exchange [8] seem to imply that they are absent in any theory of gravity based on a *symmetric* metric tensor. Apparently, such parity violations require a *propagating* torsion which the Einstein Cartan theory does not have. However, since these conclusions emerge only in the weak field approximation, they may not preclude conclusively the possibility of gravitational parity violation in Einstein Cartan theory. Admittedly, effects arising from the latter may have somewhat less of a prospect of detection in laboratory experiments (or even in astrophysical data) than those predicted phenomenologically [8]. On the other hand, they may be significant in the context of cosmology, as mentioned above.

In this paper, the problem of gauge invariant coupling of the Einstein Cartan theory to the Maxwell field is reconsidered in the light of wisdom gleaned from string theory. We focus on a situation where the source of torsion - the Kalb-Ramond antisymmetric tensor field - violates spatial parity.

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II. GAUGE INVARIANCE AND TORSION

As already stated, the electromagnetic field strength is defined by

$$F_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu \quad (1)$$

where the covariant derivative is to be written as

$$D_\mu A_\nu = \partial_\mu A_\nu + \Gamma_{\mu\nu}^\rho A_\rho, \quad (2)$$

where the affine connection Γ includes torsion. Thus the expression for $F_{\mu\nu}$ in our case turns out to be

$$F_{\mu\nu} = \partial_{[\mu} A_{\nu]} - 2T_{\mu\nu}^\rho A_\rho \quad (3)$$

which is obviously not invariant under the standard $U(1)$ electromagnetic gauge transformation $\delta A_\mu = \partial_\mu \omega$. Clearly, this is quite unacceptable, as the field strengths are measurable quantities even in a curved spacetime with torsion. Now, the torsion tensor $T_{\mu\nu}^\rho$ - a purely geometric quantity like curvature must be gauge invariant. To see why this must be so, consider the behaviour of a charged scalar field under transport along a closed curve, as essentially given by

$$[\nabla_\mu, \nabla_\nu] \phi = T_{\mu\nu}^\rho \nabla_\rho \phi + F_{\mu\nu} \phi, \quad (4)$$

where, $\nabla_\mu \equiv D_\mu + A_\mu$. In (4), the *lhs* transforms exactly like ϕ under $U(1)$ gauge transformations, as does the second term on the *rhs* as $F_{\mu\nu}$ is expected to be gauge invariant. It follows that the torsion tensor in the first term on the *rhs* must therefore be gauge invariant. Thus, as already mentioned earlier, it is not possible to use it to compensate for the loss of gauge invariance manifest in (3).

If indeed no other non-gravitational fields are available, a gauge invariant coupling of the Maxwell field to torsion might well nigh be impossible. The situation is far more hopeful if there exists yet another non-gravitational field, possibly massless, to function as the source of the torsion. Within the option of bosonic fields, the Kalb-Ramond (KR) antisymmetric second rank tensor field $B_{\mu\nu}$ appears as a possible candidate. Indeed, a contact coupling between such a field with the electromagnetic field strength tensor, has already been proposed [10]. Now, since $B_{\mu\nu}$ is a massless antisymmetric field, it is expected to be a gauge connection, as indeed it is, with the following gauge transformation $\delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}$ which leaves its field strength $H_{\mu\nu\lambda} \equiv \partial_{[\mu} B_{\nu\lambda]}$ gauge invariant. Unfortunately, in [10] (and other earlier works on this theme), this invariance has not been sufficiently respected.

There is yet another reason to consider the massless KR field: it is an inescapable part of the massless spectrum of any critical string theory, appearing upon compactification to standard four dimensional spacetime [11]. The gauge invariance mentioned above is of course well-known in the string context.

III. GAUGE INVARIANT EINSTEIN-CARTAN-MAXWELL-KALB-RAMOND COUPLING

The first step is to realize that in order to obtain a coupling that is invariant under both electromagnetic and Kalb-Ramond gauge transformations, the KR tensor potential must be endowed with a non-trivial electromagnetic gauge transformation property, and the KR field strength must be modified with the addition of an electromagnetic Chern Simons three tensor. One of the motivations of the latter comes from string theory [11], where the KR 3-form H is modified by addition of Yang-Mills and gravitational Chern Simons 3-forms to ensure that the quantum theory is anomaly free. Thus, the modified KR field strength three-tensor in our case is defined as

$$\tilde{H}_{\mu\nu\lambda} \equiv H_{\mu\nu\lambda} + \frac{1}{3} A_{[\mu} F_{\nu\lambda]}. \quad (5)$$

This modified tensor $\tilde{H}_{\mu\nu\lambda}$ is gauge invariant under standard $U(1)$ gauge transformations, provided we stipulate that the KR potential transforms under $U(1)$ gauge transformations as $\delta B_{\mu\nu} = -\omega F_{\mu\nu}$. We have of course used the standard Bianchi identities for the Maxwell field involving the Christoffel connection. Further, the Maxwell field is assumed to be left invariant under KR gauge transformations.

We now propose the following action for a manifestly gauge invariant Einstein-Cartan-Maxwell-Kalb-Ramond coupling,

$$S = \int d^4x \sqrt{-g} [R(g, T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} \tilde{H}_{\mu\nu\lambda} \tilde{H}^{\mu\nu\lambda} + T^{\mu\nu\lambda} \tilde{H}_{\mu\nu\lambda}] \quad (6)$$

where R is the scalar curvature, defined as $R = R_{\alpha\mu\beta\nu}g^{\alpha\beta}g^{\mu\nu}$. $R_{\alpha\mu\beta\nu}$ is the Riemann-Christoffel tensor:

$$R_{\mu\nu\lambda}^{\kappa} = \partial_{\mu}\Gamma_{\nu\lambda}^{\kappa} - \partial_{\nu}\Gamma_{\mu\lambda}^{\kappa} + \Gamma_{\mu\sigma}^{\kappa}\Gamma_{\nu\lambda}^{\sigma} - \Gamma_{\nu\sigma}^{\kappa}\Gamma_{\mu\lambda}^{\sigma} \quad (7)$$

The torsion tensor $T_{\mu\nu\lambda}$ is an auxiliary field in eq. (6), obeying the constraint equation

$$T_{\mu\nu\lambda} = \tilde{H}_{\mu\nu\lambda} . \quad (8)$$

Thus, the augmented KR field strength three tensor plays the role of the spin angular momentum density [2]. Substituting the above equation in (6) and varying with respect to $B_{\mu\nu}$ and A_{μ} respectively, we obtain the equations

$$D_{\mu}^S \tilde{H}^{\mu\nu\lambda} = 0 \quad (9)$$

and

$$D_{\mu}^S F^{\mu\nu} = \tilde{H}^{\mu\nu\lambda} F_{\lambda\mu} , \quad (10)$$

where, D^S is the covariant derivative using the Christoffel connection. Clearly, these equations of motion are manifestly gauge covariant under both gauge transformations. The interaction term thus has the structure

$$S_{int} = \int d^3x \sqrt{-g} H^{\mu\nu\lambda} A_{\mu} F_{\nu\lambda} . \quad (11)$$

Such a structure has been proposed earlier [4] on quite different grounds to solve the problem of gauge invariant Einstein-Cartan-Maxwell couplings. Since the KR three tensor is Hodge-dual to the derivative of a spinless field ϕ , so that, after a partial integration, one obtains,

$$S_{int} = \frac{1}{2} \int d^4x \phi F_{\mu\nu} {}^*F^{\mu\nu} , \quad (12)$$

where, ${}^*F^{\mu\nu} \equiv \epsilon^{\mu\nu\lambda\sigma} F_{\lambda\sigma}$. Here, we have noted the fact that

$$\frac{1}{\sqrt{-g}} \partial_{\nu} (\sqrt{-g} {}^*F^{\mu\nu}) = D_{\mu}^S {}^*F^{\mu\nu} = 0 \quad (13)$$

by the Maxwell Bianchi identity.

In standard string theory inspired applications to high energy phenomenology, the spinless field is considered to be a pseudoscalar - prospectively an axion. Here we depart from this interpretation. In ref. [6], it has been shown how to construct parity violating actions in pure gravity as well as with matter couplings, by starting with a parity violating torsion tensor and using the minimal coupling prescription. Although the above construction of torsion coupling of the electromagnetic field using the KR antisymmetric tensor field is not minimal in the sense of [6], parity violation can easily be incorporated here as well. Indeed, it has already been noted [7] that the interaction (12) above induces parity violation for the electromagnetic field, provided ϕ is a *scalar* field. However, such a proposal is in need of a firmer theoretical underpinning which our derivation provides. At the level of the antisymmetric KR field, the proposal that ϕ is a scalar may be interpreted as stipulating that KR field has the wrong parity, as in ref. [6]. Notice that the crucial ingredient in inducing gauge invariant parity violating couplings of the electromagnetic field has been the augmentation of the KR field strength by the Chern Simons term.

IV. DISCUSSION AND CONCLUSIONS

Two issues have been addressed in the foregoing : that of coupling torsion to the Maxwell field in a way that respects all Abelian gauge symmetries, and that of using such a coupling to obtain gauge invariant parity violation for the electromagnetic field. The generic Lagrangian density as depicted in (12) has been proposed earlier [3] as a counterexample to the Einstein Equivalence Principle, where the scalar field, however, is stipulated to be a function of the gravitational fields alone. The observability of the consequent rotation in the polarization plane of synchrotron radiation from distant sources has also been explored earlier [5] and found to be insignificant. An action similar to, although not identical to, (12) giving a gauge invariant coupling of the Maxwell field to the Einstein Cartan system has also been considered [4]. However, in the latter work, little is said about the possible origin of the so-called torsion

potential. In our opinion, our work ties up the loose ends of earlier approaches into one consistent framework which could in principle lead to observable predictions of gravitational parity violation.

In the recent past, some analysis of data on synchrotron radiation from distant radio galaxies has led to claims [12], albeit a bit controversial [13], that the rotation of the plane of polarization of the radiation has indeed been observed. If these claims are valid, they could provide a testing ground for our proposals here. Indeed, in a somewhat related approach [14], a massive fermion is coupled to a background torsion and integrated to produce a one-loop effective action of the type (12). However, regularization issues pertaining to the axial anomaly do not seem to have been adequately addressed in this work. If the theory is rendered completely free of gauge and gravitational anomalies, such effects may actually even disappear. Furthermore, the background torsion used in the paper has uncertain origins. In contrast, our augmentation of the KR field strength is motivated by the requirement of anomaly freedom in string theory and perhaps is therefore more consistent.

In this context, it may be noted that the augmentation as given in (5) is actually incomplete, since the Lorentz Chern Simons three form has not been added. This is necessitated by the requirement of freedom from local Lorentz anomalies [11]. It is not difficult to show that the addition of such terms to the \tilde{H} in (5) would generate additional parity violating couplings of the type

$$S_{int} = \int d^4x \phi \epsilon^{\mu\nu\rho\lambda} R_{\mu\nu\eta\sigma} R_{\rho\lambda}{}^{\eta\sigma} \quad (14)$$

which might show up in the polarization asymmetry of gravitational waves [7].

Finally we remark that any observation of the effects that we have alluded to here can be construed to imply the existence of spacetime torsion, albeit locally in spacetime, at least in the earliest epoch.

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